LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **NOVEMBER 2013**

MT 6603/MT 6600 - COMPLEX ANALYSIS

Date : 05/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

PART-A

Answer ALL questions: **(10 x2=20)**

1. Define the continuity of a function of a complex variable.
2. Show that $u = x^4 - 6x^2y^2 + y^4$ is harmonic.
3. Prove that $\left|\int_c f(z)dz\right| = ML$ where $M = \max\{|f(z)| / z \in c\}$ and L is the length of c.
4. State Morera's theorem.
5. Find the zeros of $f(z) = \frac{z^3 - 1}{z^3 + 1}$. .
6. Define removable and essential singularities.
7. Define residue of a function at a point.
8. State the Argument principle.
9. Find the invariant points of the transformation $w = \frac{-(2z + 4i)}{iz + 1}$.
10. Define a bilinear transformation.

PART – B

Answer any FIVE questions: **(5x8=40)**

11. Show that the function $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2 + y^4} & if\ z \neq 0 \\ 0 \quad if\ z \neq 0 \end{cases}$ is not differentiable at z = 0.
12. Show that $u = \log\sqrt{x^2 + y^2}$ is harmonic and determine its conjugate.
13. State and prove Liouville's theorem.
14. Expand $f(z) = ze^{2z}$ in a Taylor's series about the point z = -1.
15. Classify the singularity of the function $f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$.
16. State and prove Residue theorem.

17. Show that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.

18. Find the bilinear transformation which maps the points z = 0,-i,-1 into the points w = i, 1, 0 respectively.

PART - C

Answer any TWO questions: **(2x20=40)**

19. a) Derive Cauchy Riemann equations in polar coordinates,

b) If f(z) is an analytic function show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. (12 + 8)

20. a) State and prove Cauchy's integral formula.

b) Using Cauchy's integral formula to evaluate $\int_C \frac{e^z}{(z+2)(z+1)^2}\,dz$ where C is $|z| = 3$. (12+8)

21. a) State and prove Laurent's theorem.

b) Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a laurent's series valid for (i) $|z| < 1$ (ii) $1 < |z| < 2$

(iii) $|z| > 2$. (12+8)

22. a) Using contour integration along the unit circle , evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$.

b) Prove that any bilinear transformation which maps the unit circle $|z| = 1$ onto the unit circle $|w| = 1$ can be written in the form $w = e^{i\lambda}\left(\frac{z-\alpha}{\overline{\alpha}z-1}\right)$ where λ is real number. (12+8)

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